Optimal Bi-Valued Auctions

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Abstract

We investigate *bi-valued* auctions in the digital good setting and construct an explicit polynomial time deterministic auction. We prove an unconditional tight lower bound which holds even for random superpolynomial auctions. The analysis of the construction uses the adoption of the finer lens of *general competitiveness* which considers additive losses on top of multiplicative ones. The result implies that general competitiveness is the right notion to use in this setting, as this optimal auction is uncompetitive with respect to competitive measures which do not consider additive losses.

1   introduction

Marketing a digital good may suffer from a low revenue due to incomplete knowledge of the marketer. Consider, for example, a major sport event with some 10^8 potential TV viewers. Assume further that every potential viewer is willing to pay \$10 or more to watch the event, and that no more than 10^6 are willing to pay \$100 for that. If the concessionaire will charge \$1 or \$100 as a fixed pay per view price for the event, the overall collected revenue will be \$$10^8$ at the most. This is worse than the revenue that can be collected, having known the valuations beforehand.

This lack of knowledge motivates the study of *unlimited supply, unit demand, single item* auctions. Goldberg et al. [11] studied these auctions; in order to obtain a prior free, worst case analysis framework, they suggested to compare the revenue of these auctions to the revenue of optimal fixed price auctions. They adopted the *online algorithms* terminology [20] and named the revenue of the fixed price auction the *offline revenue* and the revenue of a multi price *truthful* auction, i.e., an auction for which every bidder has an incentive to bid its own value, *online revenue*. The *competitive ratio* of an auction for a bid vector b is defined to be the ratio between the best offline revenue for b to the revenue of that auction on b. The competitive ratio of an auction is just the worst competitive ratio of that auction on all possible bid vectors. For random auctions, a similar notion is defined by taking the expected revenue. If an auction has a constant competitive ratio it is said to be *competitive*. If an auction has a constant competitive ratio, possibly with some small additive loss, it is said to be *general competitive* (see Section 2 for definitions). We remark that later, Koutsoupias and Pierrakos [13] used *online auctions* in the usual context of online algorithms, but here we will stick to Goldberg et al.'s [11] notion.

This attempt to carefully select the right benchmark in order to obtain a prior free, worst case analysis was a posteriori justified when Hartline and Roughgarden defined a general benchmark for the analysis of single parameter mechanism design problems [12]. This general benchmark, which bridges Bayesian analysis [16] from economics and worst case analysis from the theory of computer science, collides for our setting with the optimal fixed price benchmark [12].

A further justification for taking the offline fixed price auction as a benchmark is the following. Although an online auction seems less restricted than the offline fixed price auction, as it can assign different players different prices, it was shown [9] that the online truthful revenue is no more than the offline revenue. In fact, there even exists a lower bound of 2.42 on the competitive ratio of any truthful online auction [10]. Note, however, that the optimal offline revenue (or more precisely the optimal fixed price) is unknown to the concessionaire in advance.

It is well known (see for example [15]) that in order to achieve truthfulness one can use only the set of *bid independent auctions*, i.e., auctions in which the price offered to a bidder is independent of the bidder's own bid value. Hence, an intuitive auction that often comes to one's mind is the *Deterministic Optimal Price (DOP) auction* [2, 9, 19]. In this auction the mechanism computes and offers each bidder the price of an optimal offline auction for all other bids. This auction preforms well on most bid vectors. In fact, it was even proved by Segal [19] that if the input is chosen uniformly at random, then this auction is asymptotically optimal. For a worst case analysis, however, it preforms very poor. Consider, for example, an auction in which there are n bidders and only two possible bid values: 1 and h, where $h \gg 1$. We denote this setting as *bi-valued* auctions. Let n_h be the number of bidders who bid h. Applying *DOP* on a bid vector for which $n_h = n/h$ will result in a revenue of n_h instead of the n revenue of an offline auction. This is because every "h-bidder" is offered 1 (since $n - 1 > h \cdot (n_h - 1)$) and every "1-bidder" is offered h (since $n - 1 < h \cdot n_h$). Here an "h-bidder" refers to a bidder that bids h and a "1-bidder" refers to a bidder that bids 1. Therefore, the competitive ratio of *DOP* is unbounded. Similar examples regarding the performances of *DOP* in the bi-valued auction setting appeared already in Goldberg et al. [9], and in Aggarwal et al. [2].

Goldberg et al. [11] showed that there exist random competitive auctions. Other works with different random competitive auctions, competitive lower bounds, and better analysis of existing auctions were presented, see for example [7, 8, 10]. For a survey see the work of Hartline and Karlin that appeared in [17, Chapter 13] (Profit Maximization In Mechanism Design). In all these works, no deterministic auction was presented. In fact, Goldberg et al. [9, 11] even proved that randomization is essential assuming the auction is symmetric (aka anonymous), i.e., assuming the outcome of the auction does not depend on the order of the input bids.

Aggarwal et al. [1,2] later showed how to construct from any randomized auction a deterministic, asymmetric auction with a factor 4 loss in the gained revenue. This result was then improved by the authors together with Wolfovitz [3], but no tight derandomization was ever presented.

1.1 Our Results

We introduce a tight deterministic auction for bi-valued auctions, with bid values $\{1, h\}$ and n bidders. We show a polynomial time deterministic auction, which guaranty a revenue of max $\{n, h \cdot n_h\}-$

$O(\sqrt{n \cdot h})$, where n_h is just the number of bidders that bid h. We then show that this bound is unconditionally optimal by showing that every auction (including a random superpolynomial one), cannot guaranty more than $\max\{n, h \cdot n_h\} - \Omega(\sqrt{n \cdot h})$. That is, there exists an auction with no multiplicative loss and with only $O(\sqrt{n \cdot h})$ additive loss, and every auction has at-least these losses. Let us note here, that if we restrict ourselves to anonymous auctions (symmetric) then we have a multiplicative loss of $\Omega(h)$ and an additive loss of $\Omega(n/h)$ over the $\max\{n, h \cdot n_h\}$ revenue of the best offline [2, 9].

The solution for this auction is build upon a new solution for a hat guessing game [4]. The connection between digital good auctions and hat guessing games was established by the work of Aggarwal et al. [1]. Here we reinforce these connections and show a connection to a different game called the majority game which was studied by Doerr [5] and by Feige [6].

1.2 Additive Loss

Already in the work that suggested to use competitive analysis, namely Goldberg et al. [11], a major obstacle arises. It was indicated that no auction can be competitive against bids with one high value (see Goldberg et al. [9] for details). The first solution that was suggested to this problem was taking a different benchmark as the offline auction. This different benchmark was again the maximum single price auction, only now the number of winning bidders is bounded to be at-least two. The term competitive was then used to indicate an auction that has a constant ratio on every bid vector against any single price auction that sells at-least two items. A few random competitive auctions were indeed suggested using this definition over the years, but, as noted before, no deterministic (asymmetric) auction was ever found. In fact, this was proved not to be a coincidence when Aggarwal et al. [1] showed that no deterministic auction can be competitive even on this weaker benchmark.

Given this lower bound a new solution should be considered, and indeed Aggarwal et al. [1] suggested such. The new definition suggested to generalize the competitive notion to include also additive losses on top of the multiplicative ones considered before.

We argue that our results indicate that this second approach of considering also the additive loss is more accurate, as it shows how analyzing with a finer granularity turns an uncompetitive auction to an optimal one. We elaborate on this agenda in the Discussion section (4).

2 Preliminaries

A bid-vector $b \in \{1, h\}^n$ is a vector of n bids. For $b \in \{1, h\}^n$ and $i \in [n] = \{1, 2, \ldots, n\}$ we denote by b_{-i} the vector which is the result of replacing the ith bid in b with a question mark; that is, b_{-i} is the vector $(b_1, b_2, \ldots, b_{i-1}, ?, b_{i+1}, \ldots b_n)$.

Definition 1 (Unlimited supply, unit demand, single item auction)**.** *An* unlimited supply, unit demand, single item auction *is a mechanism in which there is one* item *of unlimited supply to sell by an* auctioneer *to n bidders. The bidders place* bids *for the item according to their* valuation *of the item. The auctioneer then sets prices for every bidder. If the price for a bidder is lower than or equal to its bid, then the bidder is considered as a* winner *and gets to buy the item for its price.*

A bidder with price higher than its bid does not pay nor gets the item. The auctioneer's revenue is the sum of the winners prices.

A *truthful auction* is an auction in which every bidder bids its true valuation for the item. Truthfulness can be established through *bid-independent* auctions (see for example [15]). A *bid-independent* auction is an auction for which the auctioneer computes the price for bidder i using only the vector b_{-i} (that is, without the ith bid). Two models have been proposed for describing random truthful auctions. The first, being the *truthful in expectation*, refers to auctions for which a bidder maximizes its expected utility by bidding truthfully. The second model, the *universally truthful* is merely a probability distribution over deterministic auctions. Our results uses this second definition, however, it is known that the two models collide in this setting [14].

Definition 2 (General competitive auction). *Let $OPT(b)$ be the best fixed-price (offline) revenue for an n-bid vector b with bid values $1,\ h$. An auction A is a* general competitive auction *if its revenue (expected revenue) from every bid vector b, $P_A(b)$ is $\geq \alpha \cdot OPT(b) - o(nh)$ where α is a constant not depending on n or h.*

3   Bi-valued Auctions

We establish a connection between bi-valued auctions and a specific hat guessing game known as the *majority hat game*. This game was studied by Doerr [5] and later by Feige [6]. We use the new results regarding this game that appeared on [4], which enable us to solve the bi-valued auction problem optimally.

In a *majority hat game* there are n players, each wearing a hat colored red or blue. Each player does not see the color of its own hat but does see the colors of all other hats. Simultaneously, each player has to guess the color of its own hat, without communicating with the other players. The players are allowed to meet beforehand, hats-off, in order to coordinate a strategy.

It was shown that the maximum number of correct guesses the players can guaranty is no more than $\max\{n_r, n_b\} - \Omega(\sqrt{n})$, where n_r and n_b are the numbers of players with red and blue hats respectively [5, 6]. It was later proved that there exists an explicit strategy which guaranties $\max\{n_r, n_b\} - O(\sqrt{n})$ correct guesses for the players [4].

Consider *bi-valued auctions*, in which there are n bidders, each can select a value from $\{1, h\}$. The auction's revenue equals the number of bidders it offers 1 plus h times the number of bidders it offers h if indeed their value is h. Let $n_h(b)$ denotes the number of bidders who bids h in a bid vector b. Recall that the best offline revenue on vector b is $\max\{n, h \cdot n_h(b)\}$. In this section we will prove the following.

Theorem 3.1. *For bi-valued auctions with n bidders and values from $\{1, h\}$*

1. *There exists a polynomial time deterministic auction that for all bid vector b has revenue*

$$\max\{n, h \cdot n_h(b)\} - O(\sqrt{n \cdot h})$$

2. *There is no auction that for all bid vector b has revenue*

$$\max\{n, h \cdot n_h(b)\} - o(\sqrt{n \cdot h}).$$

Note that the lower bound result is unconditional and applies also for randomized superpolynomial auctions. We proceed with a proof for the upper bound in the next section and a proof for the lower bound in the adjacent one.

3.1 An Auction

We present next a solution to the bi-valued auction problem, namely we show an optimal polynomial time deterministic auction. We start again by describing a random auction. A derandomization will be built later using the same methods we presented in the former section for the hat guessing problem.

3.1.1 A Random Bi-valued Auction

For a fixed input b, let n_h be the number of h-bids in b and $n_h(i)$ be the number of h-bids in b_{-i}. Let $p'(i) = \frac{h \cdot n_h(i) - n}{h \cdot \sqrt{n_h(i)}}$. If $p'(i) \leq 0$ set $p(i) = 0$ and if $p'(i) \geq 1$ set $p(i) = 1$. Otherwise, $(0 < p'(i) < 1)$, set $p(i) = p'(i)$. The auction offers value h for bidder i with probability $p(i)$ and 1 otherwise.

Lemma 3.2. *The expected revenue of the auction described above is* $\max\{n, h \cdot n_h\} - O(\sqrt{n \cdot h})$

Proof. If $\exists i, p(i) \neq p'(i)$ then either $h \cdot n_h(i) \leq n$ so the auction will offer 1 to any 1-bidder and the revenue will be at-least $n = \max\{n, h \cdot n_h\}$, or $h \cdot n_h(i) - n \geq h \cdot \sqrt{n_h(i)}$ so every h-bidder will be offered h with probability $1 - O(1/\sqrt{n_h})$ and the expected revenue thus is at-least $h n_h \cdot (1 - 1/\sqrt{n_h}) = \max\{n, h \cdot n_h\} - O(\sqrt{n \cdot h})$. Either case our auction's revenue is $\max\{n, h \cdot n_h\} - O(\sqrt{n \cdot h})$. Assume now that $\forall i, p(i) = p'(i)$, note that in this case $|n - h \cdot n_h| = O(h\sqrt{n_h})$. The expected revenue for any bid vector with n_h bids of value h is then:

$$h \cdot n_h \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}} + n_h \cdot (1 - \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}}) + (n - n_h) \cdot (1 - \frac{h \cdot n_h - n}{h \cdot \sqrt{n_h}})$$

$$\geq h \cdot n_h \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}} + n_h \cdot (1 - \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}}) + (n - n_h) \cdot (1 - \frac{h \cdot n_h - n}{h \cdot \sqrt{n_h - 1}})$$

$$\geq h \cdot n_h \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}} + n \cdot (1 - \frac{h \cdot n_h - n}{h \cdot \sqrt{n_h - 1}}) + n_h \cdot (1 - \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}}) - n_h \cdot (1 - \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}})$$

$$= h \cdot n_h \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}} + n \cdot (1 - \frac{h \cdot n_h - n}{h \cdot \sqrt{n_h - 1}})$$

$$= h \cdot n_h \cdot \frac{h \cdot n_h - n}{h \cdot \sqrt{n_h - 1}} + n \cdot (1 - \frac{h \cdot n_h - n}{h \cdot \sqrt{n_h - 1}}) - \frac{h \cdot n_h}{\sqrt{n_h - 1}}.$$

Observe that the sum of the first two terms in the last expression above is $\max\{n, h \cdot n_h\} - O(\sqrt{n \cdot h})$. This is because $|n - h \cdot n_h| = O(\sqrt{n \cdot h})$. The third term however, can be absorbed also into the $O(\sqrt{n \cdot h})$, which completes the proof of the lemma. ∎

Hence our auction's expected revenue is within an additive loss of $O(\sqrt{n \cdot h})$ from the revenue of the best offline as promised. As noted before, a derandomization for this auction can be built using

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the same ideas appeared in the hat guessing game [4]. This derandomization produces an auction which has for the worst case only another additive loss of $O(\sqrt{n \cdot h})$ over the expected revenue of the random auction. Hence, in total, an additive loss of $O(\sqrt{n \cdot h})$ over the best offline revenue is achieved. We sketch this derandomization here for completeness.

3.1.2 Derandomization

Let $a(i) = h \cdot n_h(i) - n$ and $b(i) = h \cdot \sqrt{n_h(i)}$. The auction computes the value offered to bidder i according to the following.

1. Let $X(i) = \sum_j j$, where the sum ranges over all $j \neq i$ such that the jth bidder bids h.

2. Let $Y(i) = \sum_j 1$, where the sum ranges over all $j < i$ such that the jth bidder bids h.

3. Let $Z(i) = i + X(i) + (b(i) - 1)Y(i) \pmod{b(i)}$.

4. Offer h to bidder i if $Z(i) < a(i)$. Otherwise offer 1 to the i's bidder.

Note that for the random auction whenever $p(i) = p'(i)$ (or as stated here $a(i)/b(i) \in [0,1]$) we can define the probability that a 1-bidder will be offered 1, $p_{1,1} = (1 - \frac{h \cdot n_h - n}{h \cdot \sqrt{n_h}})$, the probability that an h-bidder will be offered 1, $p_{h,1} = (1 - \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}})$ and the probability that an h-bidder will be offered h, $p_{h,h} = \frac{h \cdot (n_h - 1) - n}{h \cdot \sqrt{n_h - 1}}$.

Lemma 3.3. *An auction that follows the above formulation gains revenue of $n_h \cdot (h \cdot p_{h,h} + p_{h,1}) - O(\sqrt{n \cdot h})$ from all h-bidders. From the 1-bidders, the auction collects $(n - n_h)p_{1,1} - O(\sqrt{n \cdot h})$.*

Proof. Let $a(1)$ be the (identical) value $a(i)$ computed by all 1-bidders. In the same manner let $b(1)$, $a(h)$, $b(h)$ be the (identical) values computed by all bidders. The lemma follows Lemma 3.2 and the following claim:

Claim 3.4.

- *For every $b(1)$ consecutive 1-bidders the auction will offer h to $a(1)$ of them and 1 to $b(1) - a(1)$*

- *For every $b(h)$ consecutive h-bidders the auction will offer h to $a(h)$ of them and 1 to $b(h) - a(h)$*

Proof. Consider the h-bidders first. Let $1 \leq i < j \leq n$ be the indices of two consecutive h-bidders. We have $i + X(i) = j + X(j)$ and $Y(j) - Y(i) = 1$. Thus $Z(j) - Z(i) = b(h) - 1 \pmod{b(h)}$. This implies that out of each $b(h)$ consecutive h-bidders, $a(h)$ will be offered h and $b(h) - a(h)$ will be offered 1.

Next consider the 1-bidders. Let $1 \leq i < j \leq n$ be the indices of two consecutive 1-bidders. We have $X(i) = X(j)$ and $Y(j) - Y(i) = j - i - 1$. Thus $Z(j) - Z(i) = j - i + (b(1) - 1)(j - i - 1) = b(1)(j - i) - b(1) + 1 \equiv 1 \pmod{b(1)}$. This implies that out of each $b(1)$ consecutive 1-bidders, $b(1) - a(1)$ are offered 1. ∎

■

It is clear that this auction can be implemented in polynomial time as claimed, hence the upper bound of Theorem 3.1 follows.

Informal Remark: A natural critic that should arise at first glance of our "complicated" suggested auction is its being "unintuitive". How can one explain/excuse suboptimal actions whenever $n_h \neq n/h$? Why not deploy DOP in these settings? Note, however, that the proposed auction does exactly the same. On most inputs it acts as the DOP and only on inputs where $n_h \approx n/h$ it deploys the "sophisticated" auction. In particular, the auction sacrifices the accuracy of results whenever for the bid vector b we have that $n \leq hn_h(b) \leq n + h\sqrt{n_h(b)}$. This "sophisticated sacrifice", however, results in turning an unbounded competitive auction into an optimal one.

Note also that the connection between bi-valued auctions and the majority game is in both directions. Thus, a solution to the bi-valued auction implies a solution to the majority game and in particular, an answer to Feige's open question [6]

3.1.3 A Lower Bound

We prove optimality of the suggested auction in the previous section. For this we prove a lower bound on the additive loss of any bi-valued auction. The lower bound is unconditional and holds also for the expected revenue of random auctions. Furthermore, the bound does not depend on the computation time needed for the auction.

Lemma 3.5. *Let A be an auction for the bi-valued $\{1, h\}$ setting and let $P_A(b)$ be A's revenue on bid vector b. Then $P_A(b)$ equals $\max\{n, h \cdot n_h\} - \Omega(\sqrt{h \cdot n})$, where b is of size n and n_h is the number of bids of value h in b.*

Proof. To prove a lower bound on the difference between the maximum offline revenue, $\max\{n, h \cdot n_h\}$, and any auction we define a distribution $\mathcal{D}$ on the possible two-values bid vectors $\{1, h\}^n$. We then show that for any deterministic auction, the expected revenue for a random bid vector b (expectation now is with respect to $\mathcal{D}$), is at most P. On the other hand, we show that the expected revenue of the offline single price (over the distribution $\mathcal{D}$) is at least $P + \Delta$, for some Δ. This implies (by standard averaging argument, see for example [21]), that for any auction, (including randomized ones), there must be some vector b for which the auction's revenue is Δ less than the fixed-price offline optimal auction.

The distribution $\mathcal{D}$ in our case is quite simple: for every bidder $i \in [n]$ independently, set $b_i = h$ with probability $1/h$ and $b_i = 1$ with probability $1 - 1/h$. Now, for every deterministic truthful auction, knowing $\mathcal{D}$, the price for every element should better be in $[h]$, otherwise there is another auction that assign prices in $[h]$ and achieves at least the same revenue for every bid vector (the one that assigns 1 for every value less than 1 and h for every value higher than h). Further, for such auction, the revenue is the sum of revenues obtained from the n bidders. Thus the expectation is the sum of expectations of the revenue obtained from the single bidders. Since for bidder i the expectation is exactly 1 (for any fixed b_i the auction must set a constant price $\alpha \in [h]$ independent

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of b_i. Hence for $\alpha > 1$, the expected revenue from bidder i is $\frac{1}{h} \cdot h = 1$, and for $\alpha = 1$ the expected value is clearly 1). We conclude that for every deterministic truthful auction as above, the expected revenue (with respect to $\mathcal{D}$), is exactly n.

We now want to prove that the expected revenue of the fixed-price offline auction, that knows b, is $n + \Omega(\sqrt{hn})$. We know, however, the exact revenue of such auction for every bid vector b. It is just $M(b) = \max\{n, h \cdot n_h(b)\}$, where $n_h(b)$ is the number of h-bids in b.

Thus the expected revenue is

$$\mathbb{E}_{\mathcal{D}}[M(b)] := \sum_{i < n/h} n \cdot \binom{n}{i} \cdot (1/h)^i \cdot (1 - 1/h)^{n-i} + \sum_{i > n/h} h \cdot i \cdot \binom{n}{i} \cdot (1/h)^i \cdot (1 - 1/h)^{n-i} \quad (1)$$
$$+ \ n \cdot \binom{n}{n/h} (1/h)^{n/h} (1 - 1/h)^{n - n/h}.$$

To estimate this sum, it is instructive to examine the following deterministic auction which we note before as DOP. On each vector b, DOP assigns value h for every bidder i for which the number of h-bids in b_{-i}, is at least n/h (we assume n/h is an integer), and 1 otherwise.

On one side, as argued before, the expected revenue of DOP with respect to $\mathcal{D}$ is

$$\mathbb{E}[P_{DOP}] = n \quad (2)$$

On the other hand, the same expression, is by definition,

$$\mathbb{E}[P_{DOP}] = \sum_{i < n/h} n \cdot \binom{n}{i} \cdot (1/h)^i \cdot (1 - 1/h)^{n-i} + \sum_{i > n/h} h \cdot i \cdot \binom{n}{i} \cdot (1/h)^i \cdot (1 - 1/h)^{n-i} \quad (3)$$
$$+ (n/h) \cdot \binom{n}{n/h} (1/h)^{n/h} (1 - 1/h)^{n - n/h}.$$

Comparing the expression in Equation (1) and Equation (3), and using Equation (2), we get:

$$\mathbb{E}_D[M(b)] = n + (n - n/h) \cdot \binom{n}{n/h} (1/h)^{n/h} (1 - 1/h)^{n - n/h}.$$

Hence we conclude that the difference in expectation between offline revenue $\mathbb{E}_D[M(b)]$ and the expected revenue on any deterministic auction, which is n, is,

$$\mathbb{E}_D[M(b)] - n = n(1 - 1/h) \cdot \binom{n}{n/h} (1/h)^{n/h} (1 - 1/h)^{n - n/h}.$$

By Stirling's approximation we know that

$$\binom{n}{n/h} = \Theta\left(\frac{\sqrt{h/n}}{\sqrt{(1 - 1/h)}(1/h)^{n/h} (1 - 1/h)^{n - n/h}}\right).$$

Therefore, the additive loss is at least $\Omega(\sqrt{h \cdot n})$ as claimed. ∎

4   Discussion

Bi-valued auctions appeared in several works, such as [2, 9, 18]. We present here a connection between these auctions and a certain *hat guessing game* [4–6]. The new optimal solution for this puzzle results in an optimal deterministic auction for bi-valued auctions. Surprisingly, the establishment of the tight lower bound for these auctions, involves with analyzing the *DOP*, the deterministic optimal auctions for i.i.d. inputs.

Our recent general derandomization [3] suffers from an additive loss of $\widetilde{O}(h\sqrt{n})$ over the expected revenue of a random auction. Aggarwal et al. [2] proved that every deterministic auction will suffer from an additive loss over the *best offline* auction, hence did not rule out exact derandomizations. We showed here that *every* auction (including a random one) suffers from an additive loss of $\Omega(\sqrt{nh})$ over the best offline. Clearly, our understanding of the additive loss is not complete yet and needs some further investigation.

Further research should ask whether there exists more cases of exact derandomization? Is there a general exact derandomization? Another interesting future direction, noticing that the connection between truthful auctions and hat guessing games was not a coincidence, is to reinforce these connection, maybe with different kind of auctions.

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